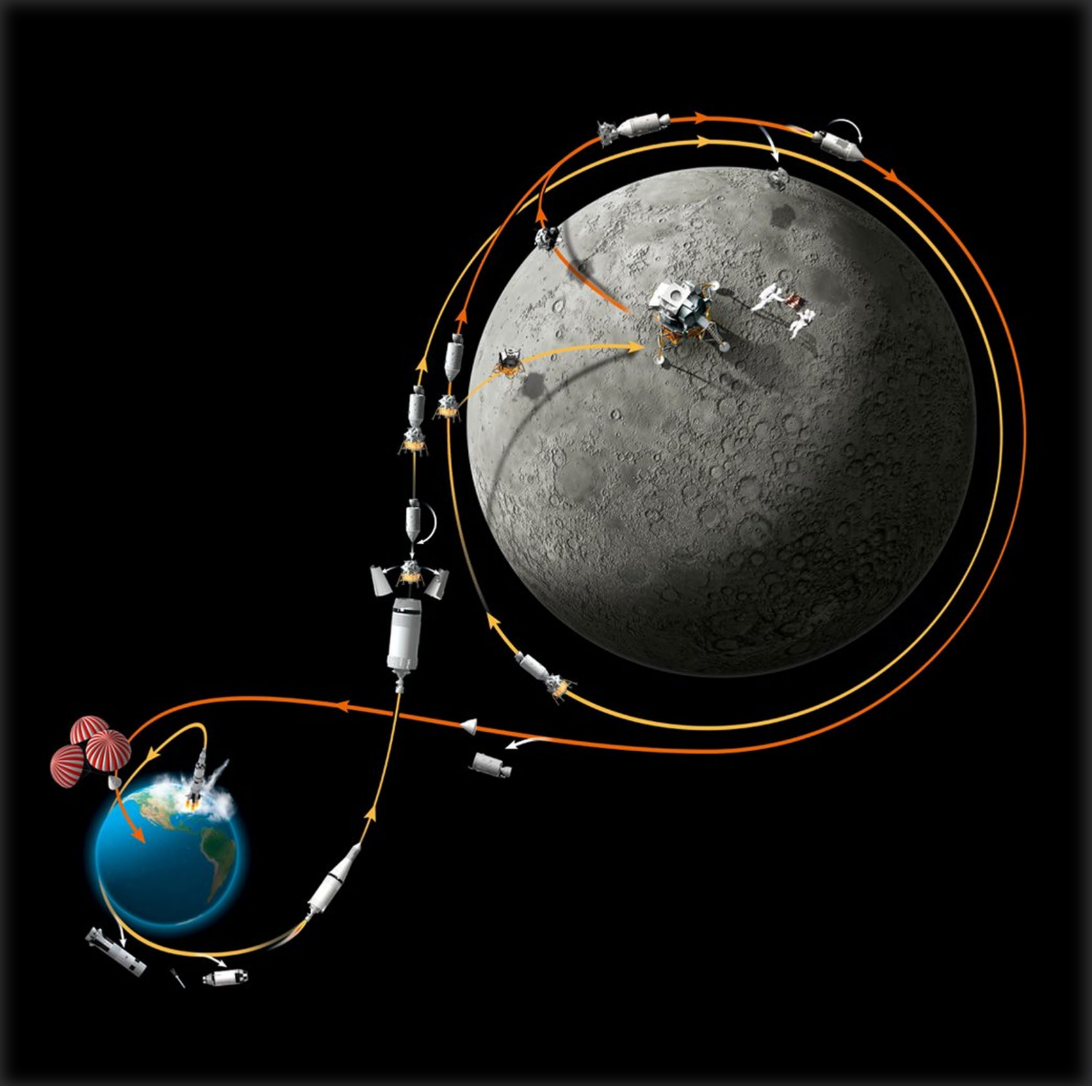
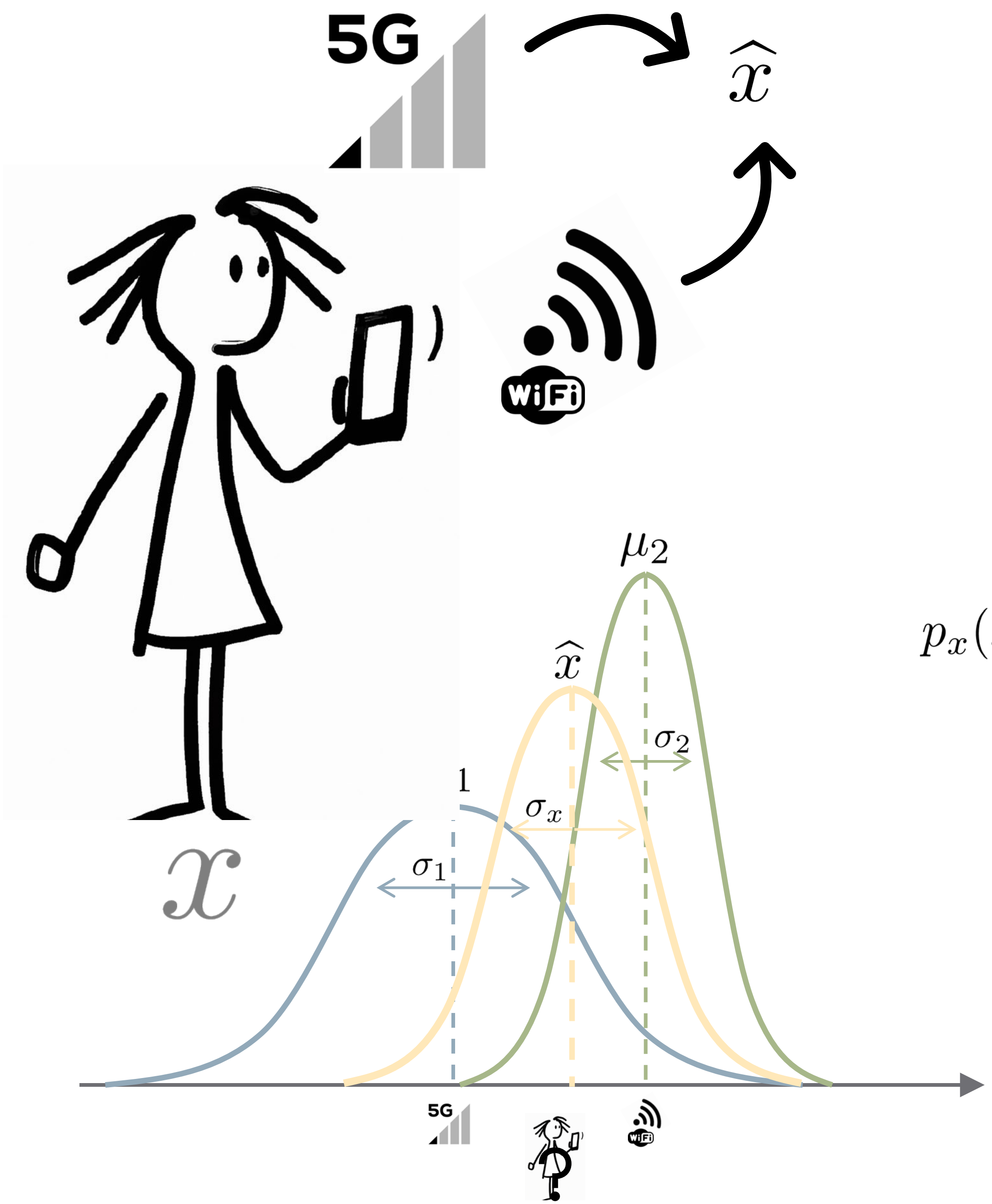
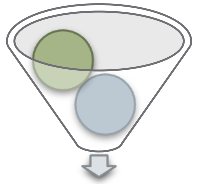


Metody i narzędzia *Big Data*

Filtr Kalmana



$$p_1(x;\mu_1,\sigma_1)$$
$$p_2(x;\mu_2,\sigma_2)$$



$$p_x(x;\underbrace{\mu_1,\mu_2}_{\widehat{x}},\underbrace{\sigma_1,\sigma_2}_{\sigma_x})$$

$$p_x(x;\mu_1,\mu_2,\sigma_1,\sigma_2) = p_1(x;\mu_1,\sigma_1)\cdot p_2(x;\mu_2,\sigma_2)$$

$$p(x|\mu) \propto p(\mu|x)\cdot p(x)$$

$$\widehat{x} = \frac{\mu_1\sigma_2^2+\mu_2\sigma_1^2}{\sigma_1^2+\sigma_2^2} \qquad \sigma_x^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}$$

jakość pomiarów

$$\widehat{x} = \frac{\mu_1\sigma_2^2 + \mu_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}\mu_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\mu_2$$

$$\widehat{x} = \mu_1 + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\big(\mu_2 - \mu_1\big)$$

nowe = stare + α przyrost

jakość pomiarów

$$\sigma_x^2 = \frac{(\sigma_1\sigma_2)^2}{\sigma_1^2 + \sigma_2^2} \quad \Rightarrow \quad \frac{1}{\sigma_x^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$$

$$\sigma_x^2 = \sigma_1^2 - \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\sigma_1^2$$

nowe = (1 - α) stare

<u>Skale pomiarowe</u>

$$\mu_1 \leftarrow C\mu_1 \quad \Rightarrow \quad \sigma_1 \leftarrow C\sigma_1$$

$$\mu_x \leftarrow C\mu_x \quad \Rightarrow \quad \sigma_x \leftarrow C\sigma_x$$

$$\boxed{\widehat{x} = \mu_1 + K\big(\mu_2 - C\mu_1\big)}$$

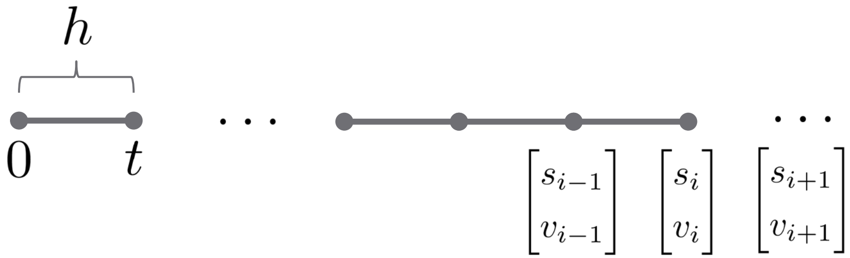
$$K = \frac{C\sigma_1^2}{(C\sigma_1)^2 + \sigma_2^2}$$

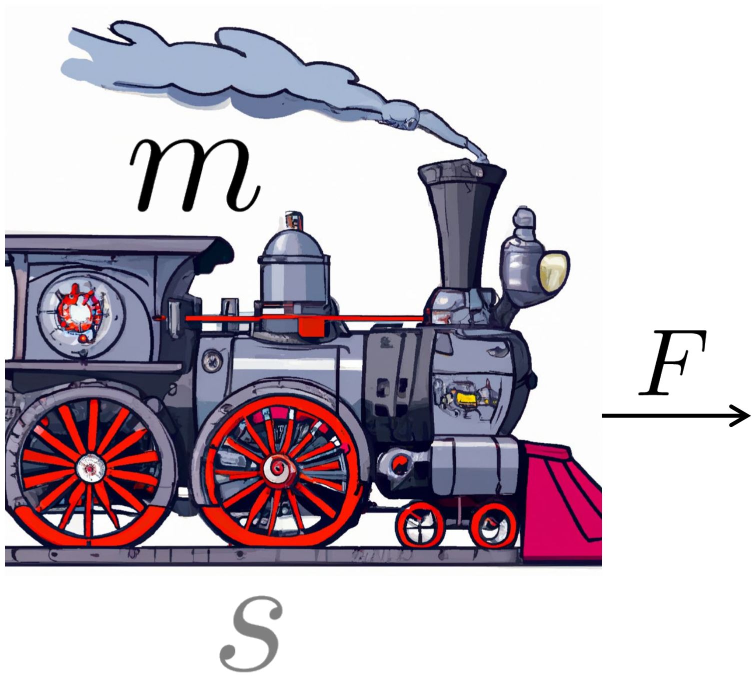
$$\boxed{\sigma_x^2 = \sigma_1^2 - KC\sigma_1^2}$$

$$a(t) = \frac{F}{m}$$

$$\begin{bmatrix} s_i \\ v_i \end{bmatrix} = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s_{i-1} \\ v_{i-1} \end{bmatrix} + \begin{bmatrix} h^2/2 \\ h \end{bmatrix} \frac{F_i}{m}$$

$$y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s_i \\ v_i \end{bmatrix}$$





<u>Proces</u>

$$\mathbf{x}_i = \mathbf{A}\mathbf{x}_{i-1} + \mathbf{B}u_i + \mathbf{w}_i \quad \sim \mathcal{N}(\mathbf{0},\mathbf{Q})$$

$$\mathbf{y}_i = \mathbf{C}\mathbf{x}_i + \mathbf{z}_i \quad \sim \mathcal{N}(\mathbf{0},\mathbf{R})$$

<u>Model</u>

$$\overline{\mathbf{x}}_i = \mathbf{A}\widehat{\mathbf{x}}_{i-1} + \mathbf{B}u_i$$

$$\overline{\mathbf{y}}_i = \mathbf{C}\overline{\mathbf{x}}_i$$

Dane: $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{Q}, \mathbf{R}$

Szukane są kolejne wartości $\widehat{\mathbf{x}}_i, \mathbf{P}_i^+$

$$\mathbf{Q} = \mathbb{E}[\mathbf{w}_i\mathbf{w}_i^{\mathrm{T}}]$$

$$\mathbf{R} = \mathbb{E}[\mathbf{z}_i\mathbf{z}_i^{\mathrm{T}}]$$

$$\mathbf{w} \perp \mathbf{z}$$

$$\mathbf{e}_i = \mathbf{x}_i - \overline{\mathbf{x}}_i$$

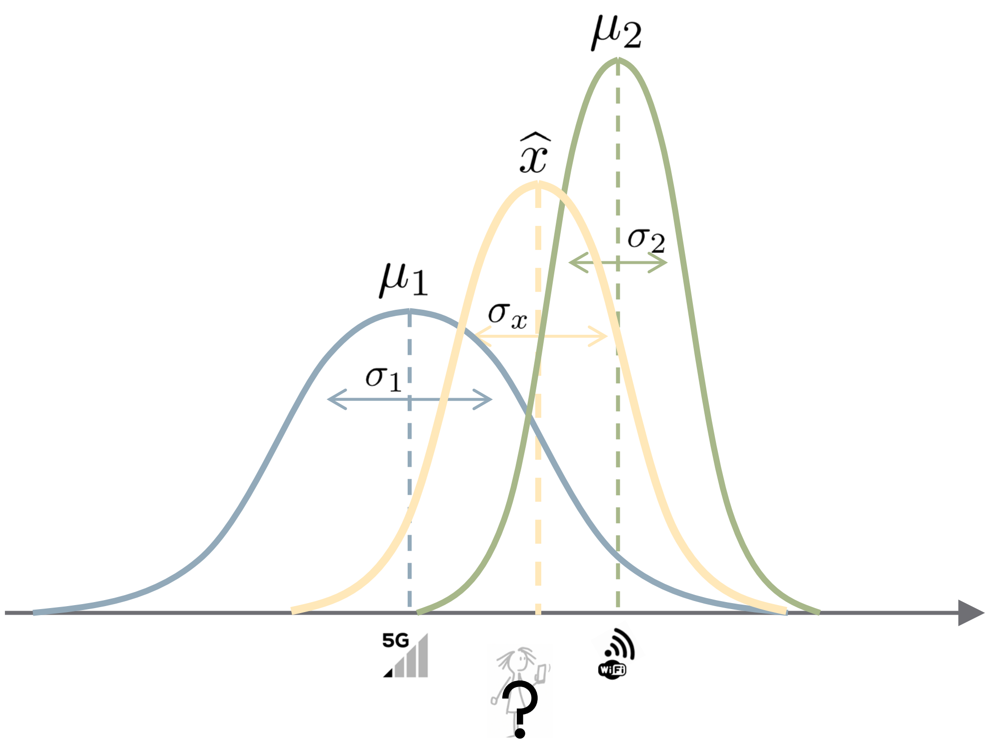
$$\widehat{\mathbf{e}}_i = \mathbf{x}_i - \widehat{\mathbf{x}}_i$$

$$\mathbf{w} \perp \mathbf{e}$$

$$\mathbf{P} = \mathbb{E}[\mathbf{e}_i\mathbf{e}_i^{\mathrm{T}}]$$

$$\mathbf{P}^+ = \mathbb{E}[\widehat{\mathbf{e}}_i\widehat{\mathbf{e}}_i^{\mathrm{T}}]$$

Fuzja pomiarów z różnych urządzeń

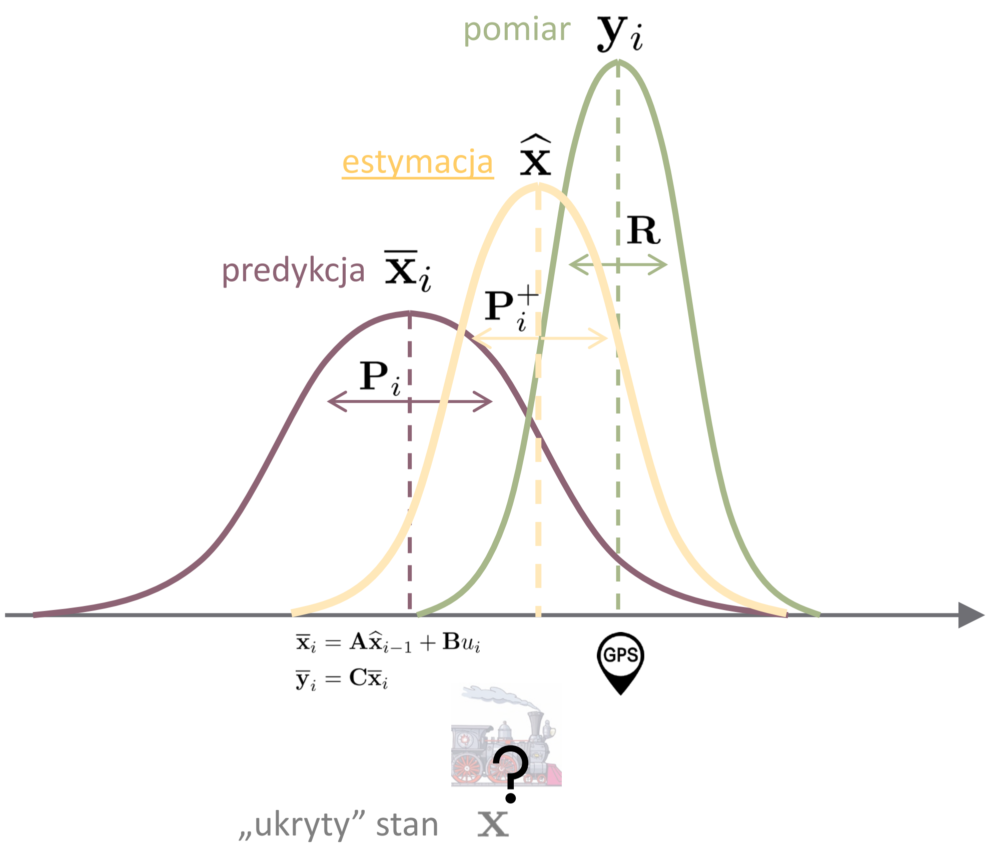


Okazuje się, że brakuje nam $\mathbf{P}_i$

$$\mathbf{P} = \mathbb{E}[\mathbf{e}_i \mathbf{e}_i^{\mathrm{T}}] = \mathbb{E}\left[(\mathbf{x}_i - \overline{\mathbf{x}}_i)(\mathbf{x}_i - \overline{\mathbf{x}}_i)^{\mathrm{T}}\right]$$

$$\mathbf{P}_i = \mathbf{A}\mathbf{P}_{i-1}^{+}\mathbf{A}^{\mathrm{T}} + \mathbf{Q}$$

Fuzja pomiaru i predykcji z modelu



Dane: $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{Q}, \mathbf{R}$

Szukane są kolejne wartości $\widehat{\mathbf{x}}_i, \mathbf{P}_i^{+}$

Wersja wielowymiarowa

$$\boxed{\widehat{x} = \mu_1 + K\big(\mu_2 - C\mu_1\big)}$$

$$K = \frac{C\sigma_1^2}{(C\sigma_1)^2 + \sigma_2^2}$$

$$\boxed{\sigma_x^2 = \sigma_1^2 - KC\sigma_1^2}$$

$$\mathbf{K} = \mathbf{P}\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{P}\mathbf{C}^{\mathrm{T}} + \mathbf{Q}\right)^{-1}$$

Założenie: dysponujemy dokładną wartością $\mathbf{K}$

$$\boxed{\widehat{\mathbf{x}} = \overline{\mathbf{x}} + \mathbf{K}\big(\mathbf{y} - \mathbf{C}\overline{\mathbf{x}}_i\big)}$$

$$\boxed{\mathbf{P}^{+} = \mathbf{P} - \mathbf{KCP}}$$

Algorytm filtru Kalmana

$$\overline{\mathbf{x}}_i = \mathbf{A}\widehat{\mathbf{x}}_{i-1} + \mathbf{B}u_i$$

$$\mathbf{P}_i = \mathbf{A}\mathbf{P}^{+}_{i-1}\mathbf{A}^{\mathrm{T}} + \mathbf{Q}$$

Predykcja

Korekcja / estymacja

$$\mathbf{K}_i = \mathbf{P}_i\mathbf{C}^{\mathrm{T}}\left(\mathbf{C}\mathbf{P}_i\mathbf{C}^{\mathrm{T}} + \mathbf{Q}\right)^{-1}$$

$$\widehat{\mathbf{x}}_i = \overline{\mathbf{x}}_i + \mathbf{K}_i\left(\mathbf{y}_i - \mathbf{C}\overline{\mathbf{x}}_i\right)$$

$$\mathbf{P}^{+}_i = \mathbf{P}_i - \mathbf{K}_i\mathbf{C}\mathbf{P}_i$$

https://filterpy.readthedocs.io/en/latest/

https://nbviewer.org/github/rlabbe/Kalman-and-Bayesian-Filters-in-Python/blob/master/table_of_contents.ipynb

The End