$$\Theta = (XX^T)^{-1} X Y^T$$

$$X_{S \times N} \qquad Y_{1 \times N}$$

$$\wr$$

$$\hat{Y}_{1 \times N}$$

podejrzane

$$Y \approx \Theta^T X$$

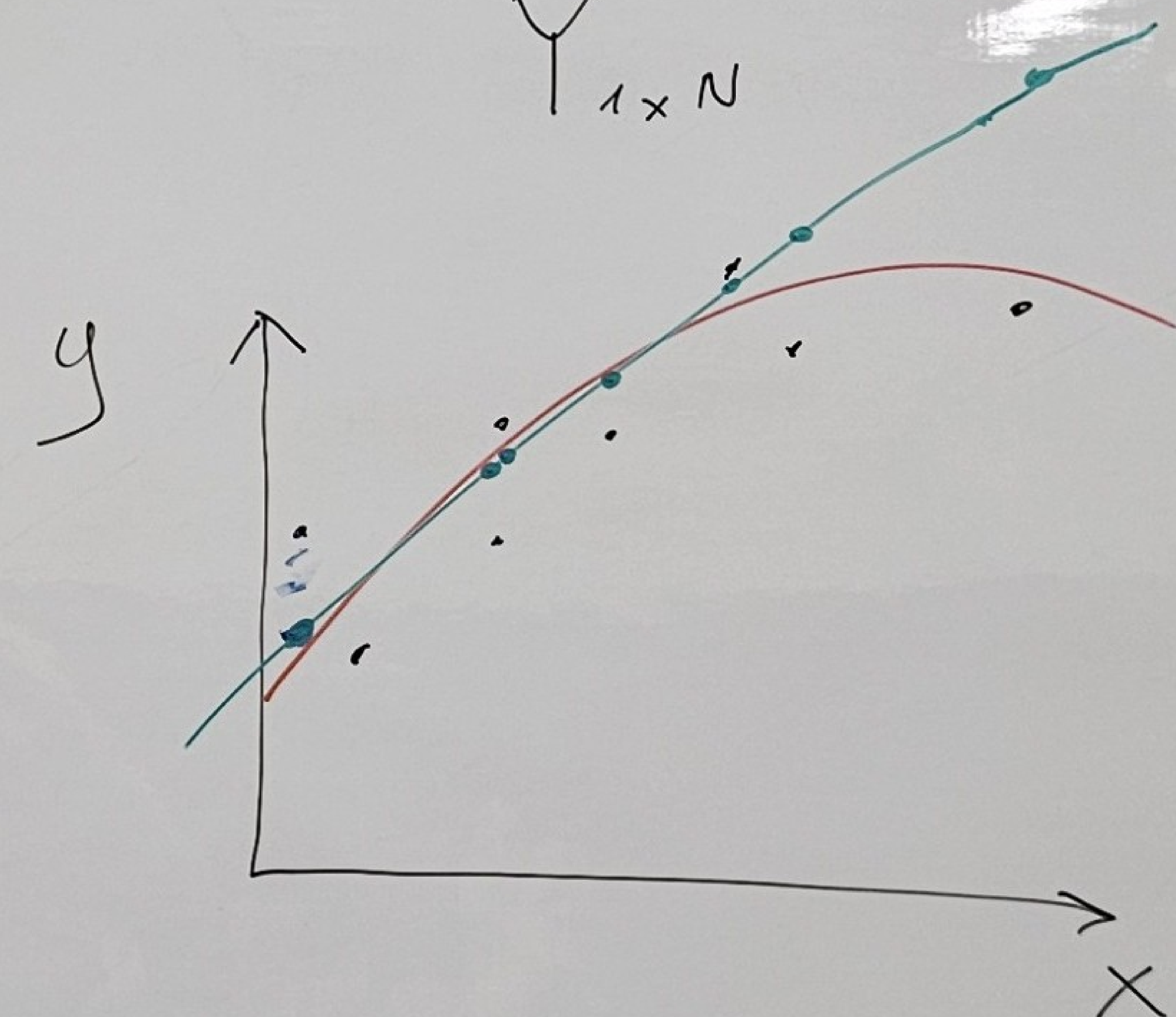
$$\hat{Y} = \Theta^T X = \phi(x;\Theta)$$

$$y = \Theta^T x$$

$$Y = [y_1 \ y_2 \ \cdots \ y_N]$$

$$| \quad | \quad |$$

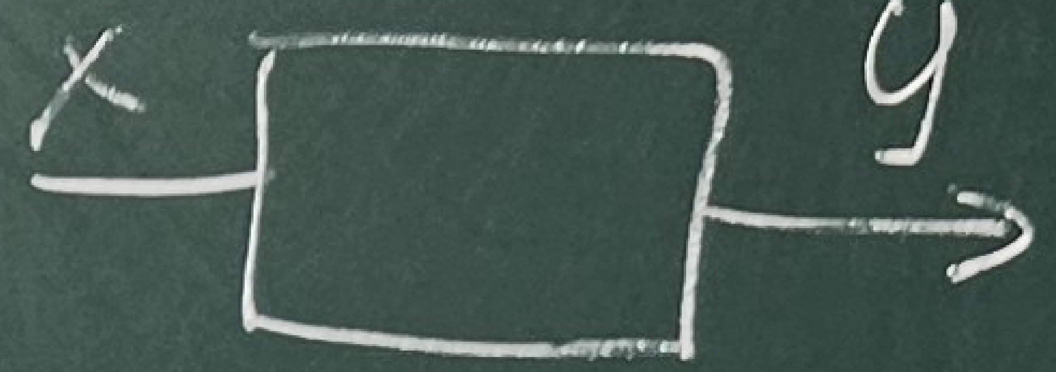
$$\hat{Y} = [\hat{y}_1 \ \hat{y}_2 \ \cdots \ \hat{y}_N]$$



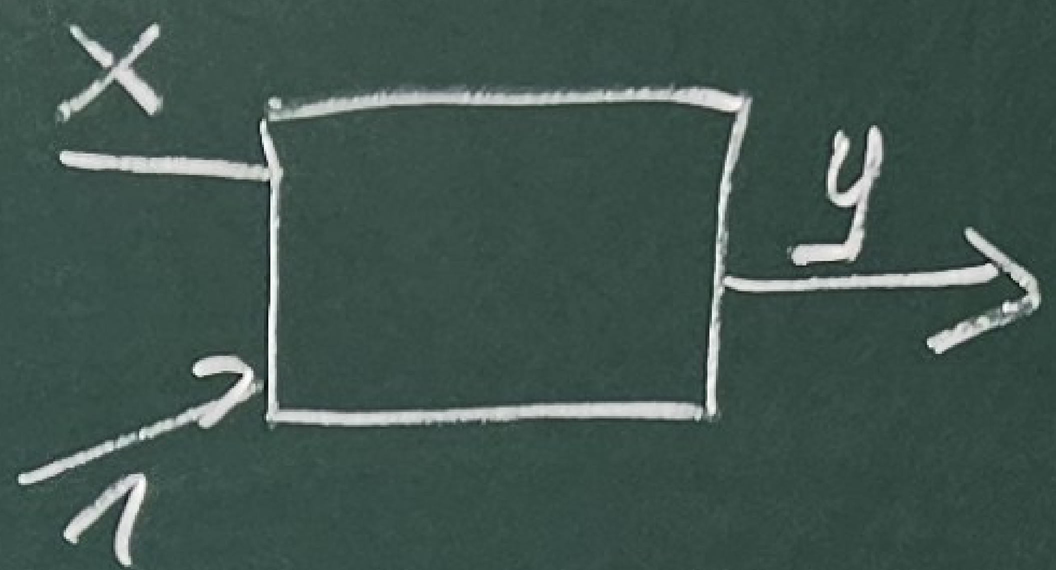

$$E = [e_1 \ e_2 \ \cdots \ e_N]$$

$$e_i = y_i - \hat{y}_i$$

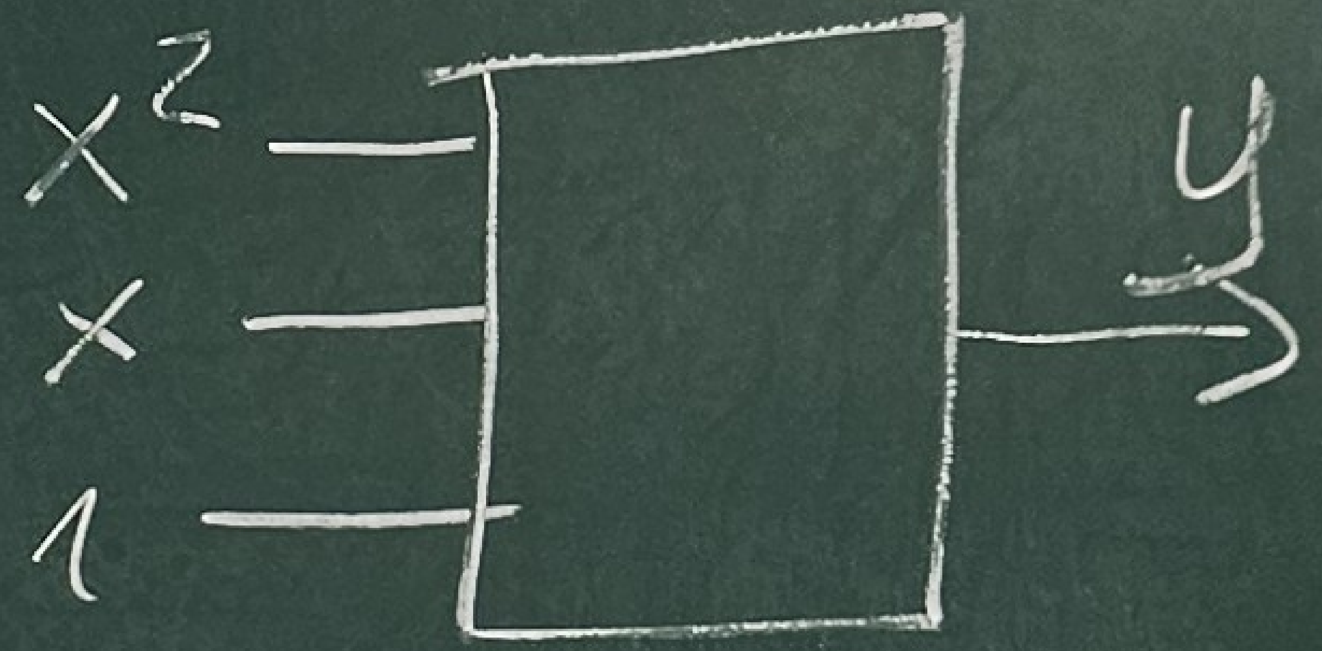
$\theta = \begin{bmatrix} a \\ b \end{bmatrix}$




$X = \begin{bmatrix} x^{(1)} \\ 1 \end{bmatrix}$




$\begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix} = \begin{bmatrix} x \\ 1 \end{bmatrix}$




dla jakich $x$, $\theta$ wyrażenie $\hat{y} = \theta^T x$ opisuje zieloną linię?

$\hat{y} = ax$ nie opisuje linii zielonej

$\hat{y} = ax + b$

to nie jest liniowa

7min - 1 osoba

8min - 2 osoby

| $x$ | $1$ | $-1$ | $2$ |
|---|---|---|---|
| $y$ | $4$ | $0$ | $3$ |

$$X = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 4 & 0 & 3 \end{bmatrix}$$

$$\tilde{X} = \begin{bmatrix} 1 & 1 & 4 \\ 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{array}{c|c|c|c} x^{(1)} & 1 & -1 & 2 \\ \hline x^{(2)} & 1 & 1 & 1 \\ \hline y & 4 & 0 & 3 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = X$$

$$\hat{y} = a_1 x^{(1)} + a_2 x^{(2)} + a_3 x^{(3)}$$

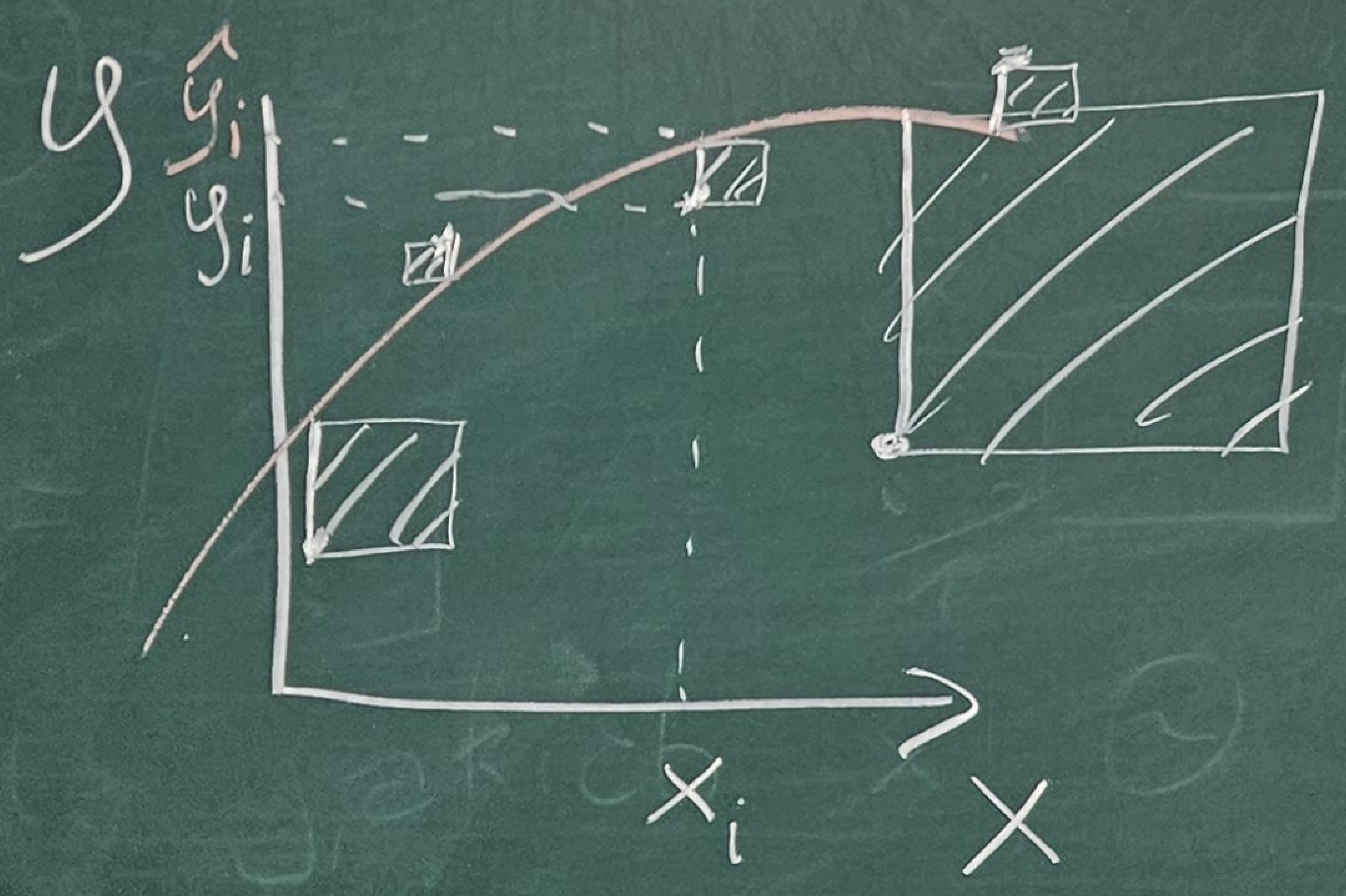
$$\hat{y} = ax^2 + bx + c$$

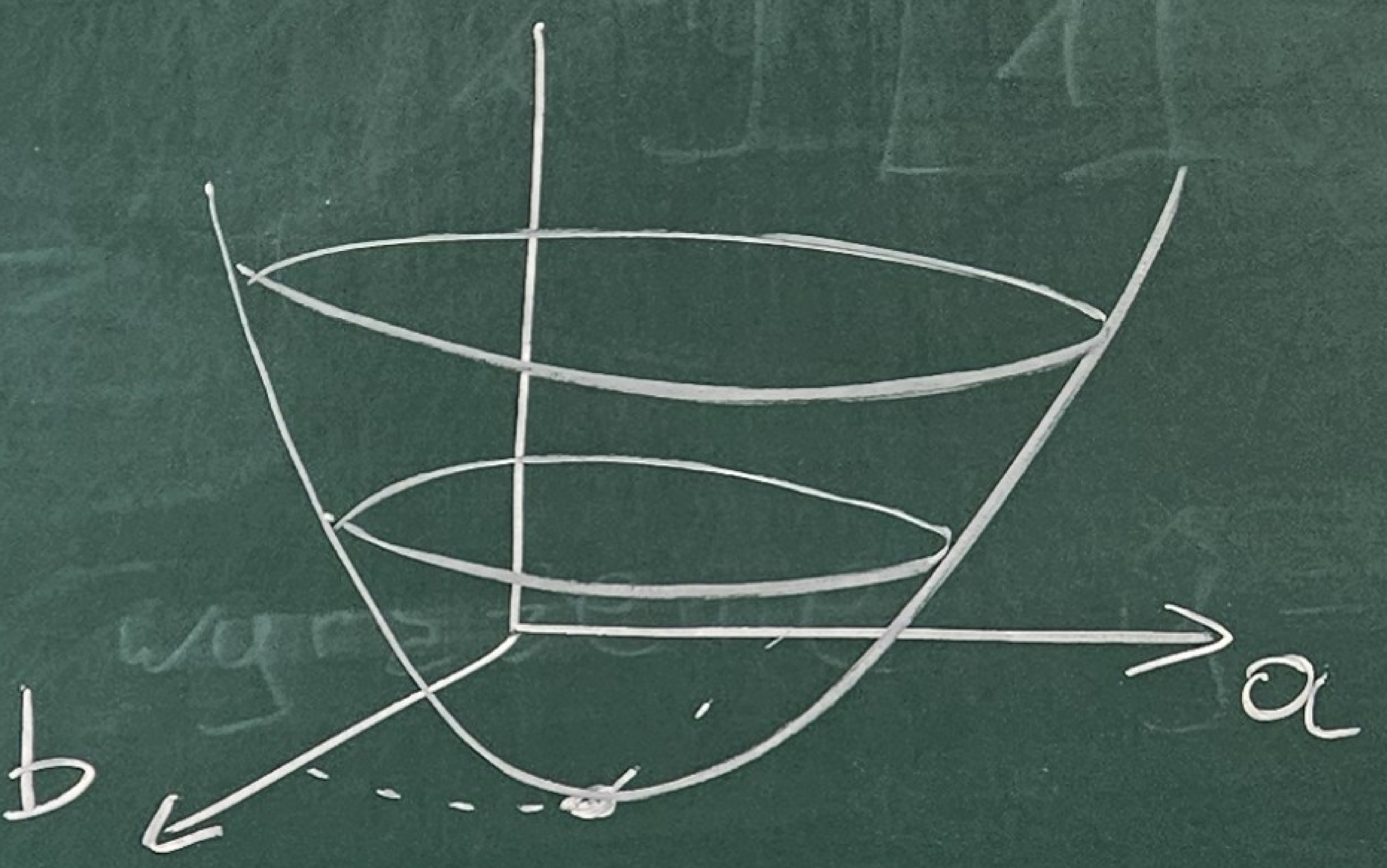
$$\hat{y} = a^T X$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$X = \begin{bmatrix} X^2 \\ X \\ 1 \end{bmatrix}$$

$$L(ax + by) = aL(x) + bL(y)$$




$$Q_1(\Theta) = \frac{1}{N}\sum_{i=1}^{N} |e_i| \rightarrow \text{tylko numer}$$

$$\frac{\partial Q}{\partial \Theta} = 0$$

$$\boxed{\min Q_2(\Theta) = \frac{1}{N}\sum_{i=1}^{N} e_i^2}$$

w domu I. Model: $\hat{y} = ax$ $\quad \frac{\partial Q_2}{\partial a}$

II Model $\quad \min Q_2$

$\hat{y} = ax + b$ $\quad \frac{\partial Q_2}{\partial a} \quad \frac{\partial Q_2}{\partial b}$

$$Q_2 = \sum_{i=1}^{N} e_i^2 = EE^T = (Y-\hat{Y})(Y-\hat{Y})^T =$$

$$\Theta = \begin{bmatrix} \Theta_1 \\ \Theta_2 \\ \vdots \\ \Theta_s \end{bmatrix}$$

$$= (Y - \Theta^T X)(Y - \Theta^T X)^T =$$

$$= (Y - \Theta^T X)(Y^T - X^T \Theta) =$$

$$Q_2 = \underbrace{YY^T}_{\mathrm{I}} - \underbrace{YX^T\Theta}_{\mathrm{II}} - \underbrace{\Theta^T XY^T}_{\mathrm{III}} + \underbrace{\Theta^T XX^T \Theta}_{\mathrm{IV}}$$

$$\frac{\partial Q_2}{\partial \Theta} = 0_s - XY^T - XY^T + 2XX^T\Theta = 0_s$$

$$\cancel{2}XY^T = \cancel{2}XX^T\Theta$$

MNK

I $\quad \frac{\partial}{\partial \Theta} YY^T = 0_S$

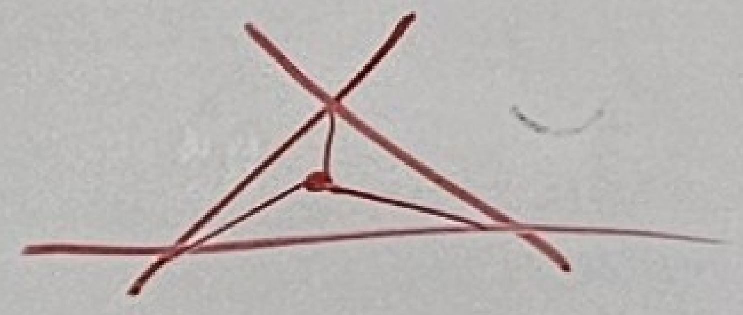
$S\left\{\begin{bmatrix}0\\0\\\vdots\\0\end{bmatrix}\right.$

II $\quad \frac{\partial}{\partial \Theta} YX^T\Theta = \left(\underset{1\times N}{Y}\,\underset{N\times S}{X^T}\right)^T = XY^T$

III $\quad \frac{\partial}{\partial \Theta} \Theta^T XY^T = XY^T$

IV $\quad \frac{\partial}{\partial \Theta} \underset{1\times S}{\Theta^T}\,\overbrace{\underset{S\times N}{X}\,\underset{N\times S}{X^T}}^{\text{sym}}\,\underset{S\times 1}{\Theta} = 2XX^T\Theta$

$(A+A^T)\Theta$



co to jest $\frac{1}{N}XX^T = \begin{bmatrix} \mathrm{cov}(x^{(1)}, x^{(1)}) & \mathrm{cov}(x^{(1)}, x^{(2)}) & \dots \\ & & \end{bmatrix}$