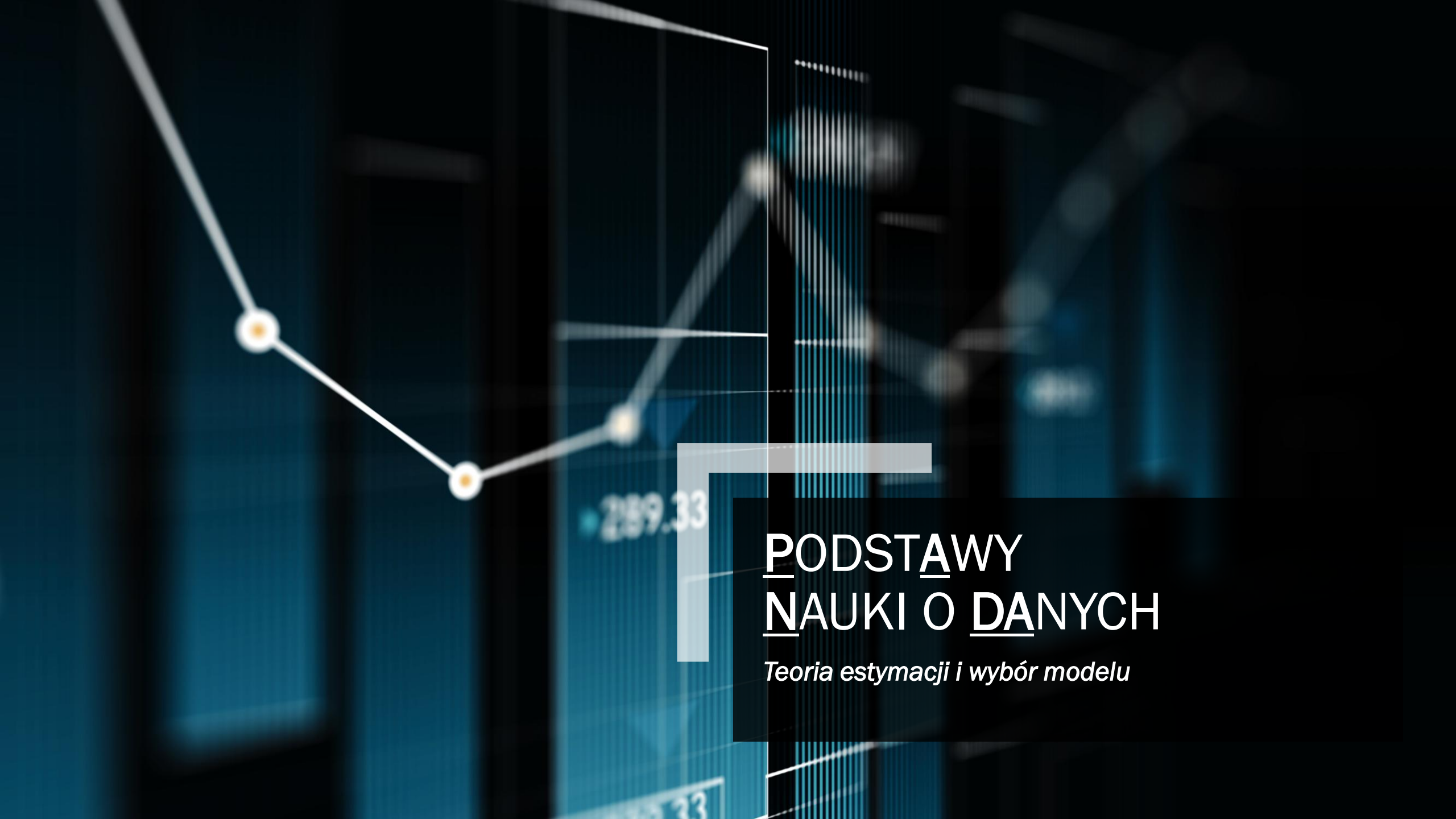
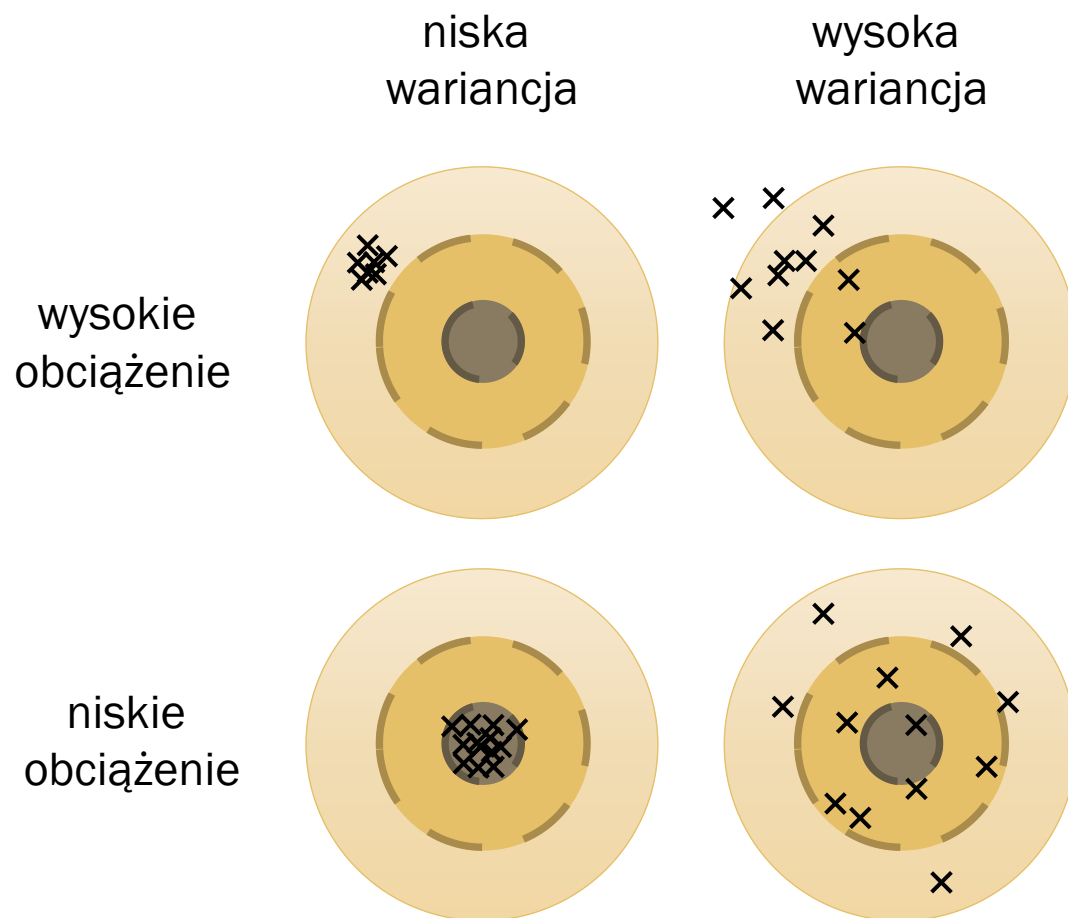




PODSTAWY NAUKI O DANYCH

Teoria estymacji i wybór modelu

Teoria estymacji – podstawy



Obciążenie (ang. *bias*)

- błąd systematyczny

Wariancja (ang. *variance*)

- błąd przypadkowy

Teoria estymacji – podstawy

$$\mathbb{E}[\spadesuit + \heartsuit] = \mathbb{E}[\spadesuit] + \mathbb{E}[\heartsuit]$$

$$\mathbb{E}[\alpha \spadesuit] = \alpha \mathbb{E}[\spadesuit], \quad \alpha \in \mathbb{R}$$

$$\mathbb{E}[\text{const.}] = \text{const.}$$

Dla niezależnych $\spadesuit, \heartsuit$ zachodzi

$$\mathbb{E}[\spadesuit \cdot \heartsuit] = \mathbb{E}[\spadesuit] \cdot \mathbb{E}[\heartsuit]$$

$$\text{Var}[\spadesuit + \text{const.}] = \text{Var}[\spadesuit]$$

$$\text{Var}[\alpha \spadesuit] = \alpha^2 \text{Var}[\spadesuit], \quad \alpha \in \mathbb{R}$$

$$\text{Var}[\text{const.}] = 0$$

Dla niezależnych $\spadesuit, \heartsuit$ zachodzi

$$\text{Var}[\spadesuit + \heartsuit] = \text{Var}[\spadesuit] + \text{Var}[\heartsuit]$$

$$\hat{\theta} = \Psi \left(\{x_i\}_{i=1}^N \right)$$

Ψ – estymator parametru (algorytm estymacji)

$$\text{Bias}[\hat{\theta}] \equiv \theta - \mathbb{E}[\hat{\theta}]$$

$$\begin{aligned} \text{Var}[\hat{\theta}] &\equiv \mathbb{E} \left[(\hat{\theta} - \mathbb{E}[\hat{\theta}])^2 \right] \\ &= \mathbb{E} \left[\hat{\theta}^2 \right] - \mathbb{E}^2[\hat{\theta}] \end{aligned}$$

Kompromis obciążenie–wariancja

$$\text{MSE}[\hat{\theta}] = \mathbb{E} \left[(\theta - \hat{\theta})^2 \right]$$

Kompromis obciążenie–wariancja

$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E} \left[(\theta - \hat{\theta})^2 \right] = \\ &= \mathbb{E} \left[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2 \right]\end{aligned}$$

Kompromis obciążenie–wariancja

$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E} \left[(\theta - \hat{\theta})^2 \right] = \\ &= \mathbb{E} \left[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2 \right] = \\ &= \mathbb{E} \left[\theta^2 \right] - \mathbb{E} \left[2\theta\hat{\theta} \right] + \mathbb{E} \left[\hat{\theta}^2 \right]\end{aligned}$$

Kompromis obciążenie–wariancja

$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E} \left[(\theta - \hat{\theta})^2 \right] = \\ &= \mathbb{E} \left[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2 \right] = \\ &= \mathbb{E} \left[\theta^2 \right] - \mathbb{E} \left[2\theta\hat{\theta} \right] + \mathbb{E} \left[\hat{\theta}^2 \right] = \\ &= \theta^2 - 2\theta\mathbb{E} \left[\hat{\theta} \right] + \mathbb{E} \left[\hat{\theta}^2 \right]\end{aligned}$$

Kompromis obciążenie–wariancja

$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E}[(\theta - \hat{\theta})^2] = \\ &= \mathbb{E}[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2] = \\ &= \mathbb{E}[\theta^2] - \mathbb{E}[2\theta\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\ &= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\ &= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] + \boxed{\mathbb{E}^2[\hat{\theta}] - \mathbb{E}^2[\hat{\theta}]}\end{aligned}$$

Kompromis obciążenie–wariancja

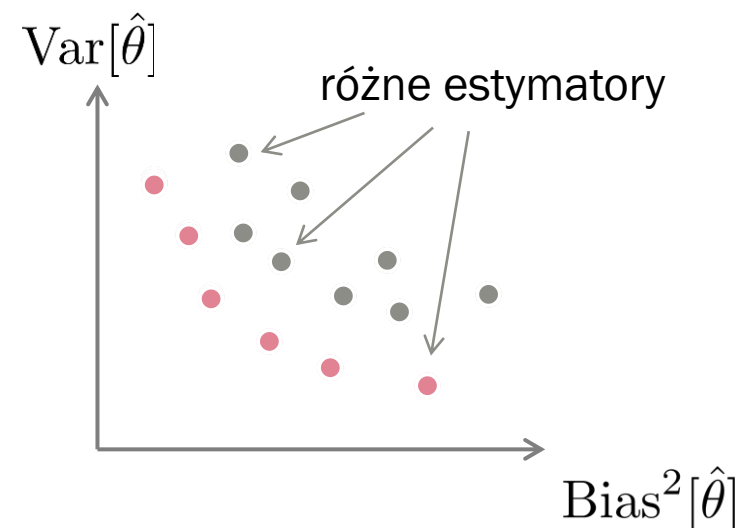
$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E}[(\theta - \hat{\theta})^2] = \\&= \mathbb{E}[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2] = \\&= \mathbb{E}[\theta^2] - \mathbb{E}[2\theta\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] + \boxed{\mathbb{E}^2[\hat{\theta}] - \mathbb{E}^2[\hat{\theta}]} = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}^2[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] - \mathbb{E}^2[\hat{\theta}]\end{aligned}$$

Kompromis obciążenie–wariancja

$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E}[(\theta - \hat{\theta})^2] = \\&= \mathbb{E}[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2] = \\&= \mathbb{E}[\theta^2] - \mathbb{E}[2\theta\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] + \boxed{\mathbb{E}^2[\hat{\theta}] - \mathbb{E}^2[\hat{\theta}]} = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}^2[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] - \mathbb{E}^2[\hat{\theta}] = \\&= \left(\theta - \mathbb{E}[\hat{\theta}]\right)^2 + \left(\mathbb{E}[\hat{\theta}^2] - \mathbb{E}^2[\hat{\theta}]\right)\end{aligned}$$

Kompromis obciążenie–wariancja

$$\begin{aligned}\text{MSE}[\hat{\theta}] &= \mathbb{E}[(\theta - \hat{\theta})^2] = \\&= \mathbb{E}[\theta^2 - 2\theta\hat{\theta} + \hat{\theta}^2] = \\&= \mathbb{E}[\theta^2] - \mathbb{E}[2\theta\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] + \underbrace{\mathbb{E}^2[\hat{\theta}] - \mathbb{E}^2[\hat{\theta}]}_{=0} = \\&= \theta^2 - 2\theta\mathbb{E}[\hat{\theta}] + \mathbb{E}^2[\hat{\theta}] + \mathbb{E}[\hat{\theta}^2] - \mathbb{E}^2[\hat{\theta}] = \\&= (\theta - \mathbb{E}[\hat{\theta}])^2 + (\mathbb{E}[\hat{\theta}^2] - \mathbb{E}^2[\hat{\theta}]) = \\&= \text{Bias}^2[\hat{\theta}] + \text{Var}[\hat{\theta}]\end{aligned}$$



Kompromis obciążenie–wariancja

ang. *bias-variance dilemma*

