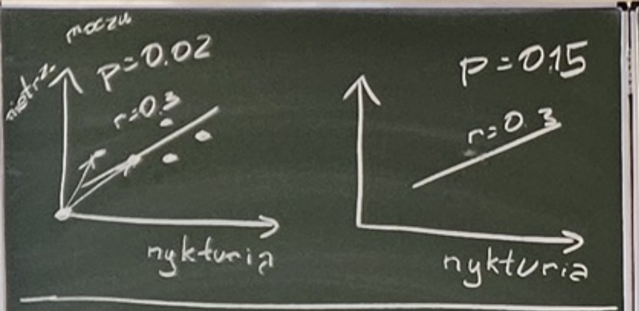


$$A\,x \qquad x \in \mathbb{R}^2$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix}$$

$$Ax = \lambda x \qquad \lambda \in \mathbb{R} \qquad v \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$Av = \lambda v$$

$$IAv - I\lambda v = O_2 \quad / \cdot I$$

$$(A - \lambda)_{2\times 2} v_{2\times 1} = O_2$$

$$(A - \lambda I)v = O_2 \Rightarrow \det(A - \lambda I) = 0$$

$$\begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ 4 & -2-\lambda \end{bmatrix}$$

$$-(1-\lambda)(\lambda+2) - 4 = 0$$

$$(\lambda-1)(\lambda+2) - 4 = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\Delta = 1 + 24 = 25 \qquad \sqrt{\Delta} = 5$$

$$\lambda_1 = -3 \qquad \lambda_2 = 2$$

$$A - \lambda_1 I = \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \qquad A - \lambda_2 I = \begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}$$

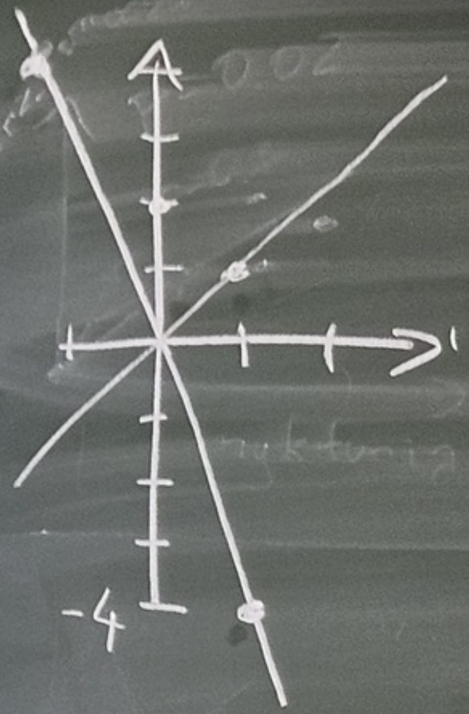
$$\begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} v_1^{(1)} \\ v_1^{(2)} \end{bmatrix}$$

$$4v_1^{(1)} + v_1^{(2)} = 0$$

$$v_1^{(2)} = -4v^{(1)}$$

$$\begin{bmatrix} v_1^{(1)} \\ -4v_1^{(1)} \end{bmatrix} = v_1^{(1)} \begin{bmatrix} 1 \\ -4 \end{bmatrix} \quad v_1$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\det A = -6$$

$$A v_1 = \lambda_1 v_1$$

$$A v_2 = \lambda_2 v_2$$

W domu przesledzić

$$\begin{bmatrix} A v_1 & A v_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 \end{bmatrix}$$

$$A \begin{bmatrix} v_1 & v_2 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$AP = P\Lambda$$

$$\boxed{A = P \Lambda P^{-1}}$$