# Computer Science

Jerzy Świątek

# Systems Modelling and Analysis

*Choose yourself and new technologies*

## L.2b. Determination of the plant parameters

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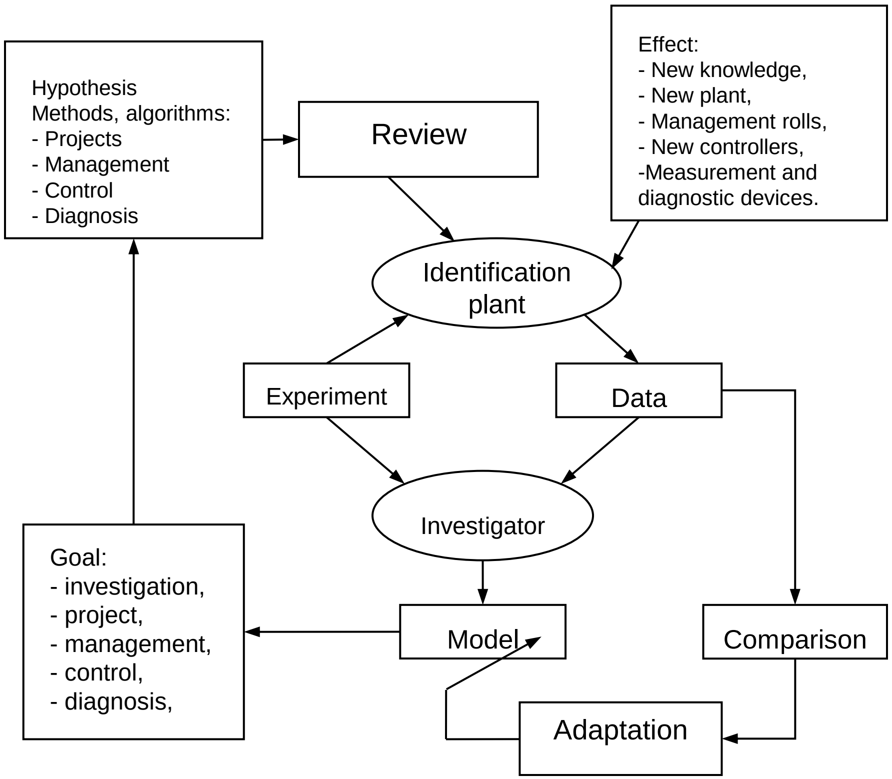
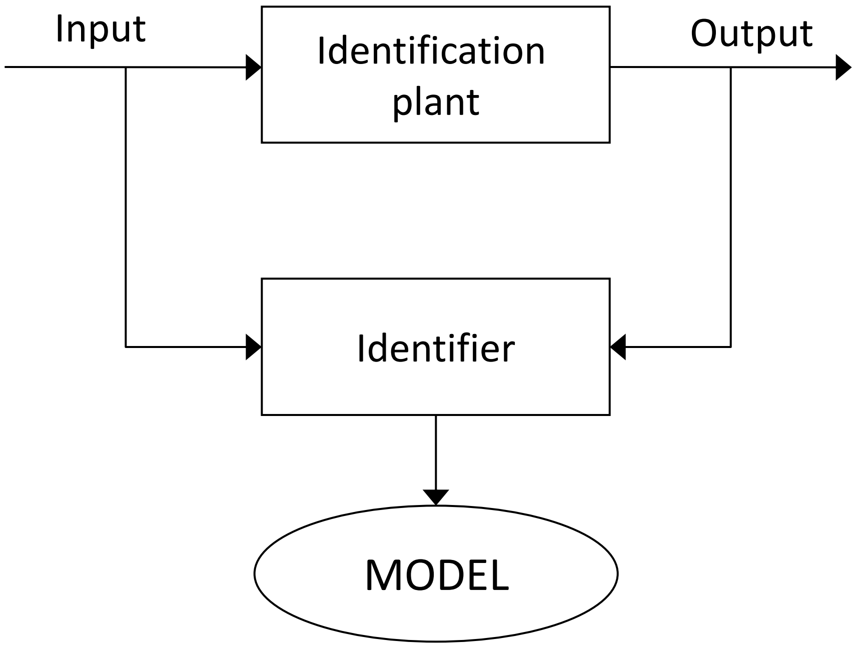
# Model in the systems research

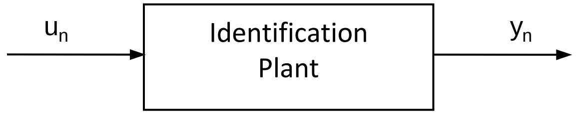


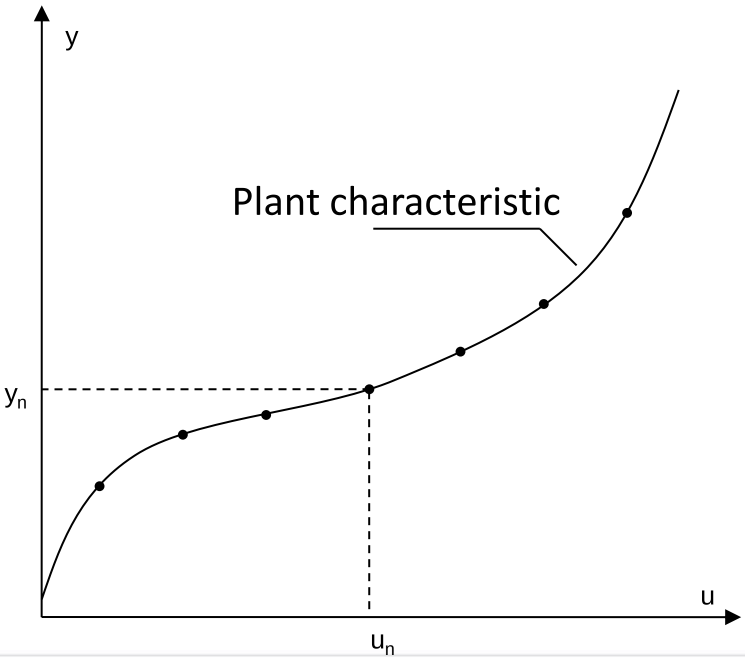
# Identification task (1)



# Ad.2. Determination of the class of model (2)

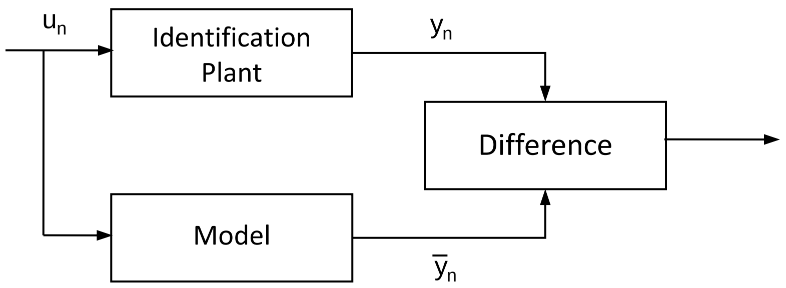
- Plant in the class of model

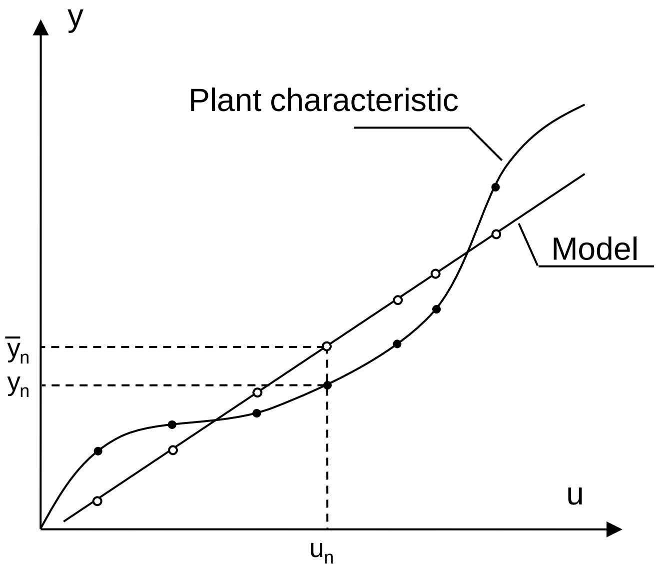


# Ad.2. Determination of the class of model (3)

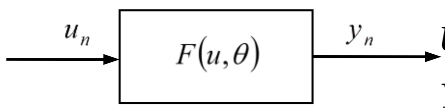
- Choice of the best model



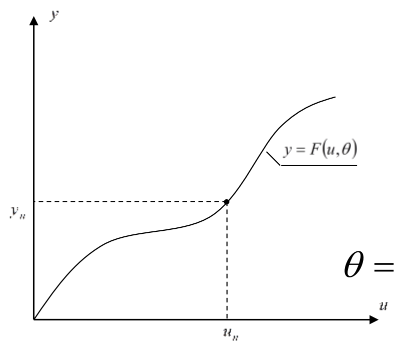



## Plant in the class of model

Deterministyczny



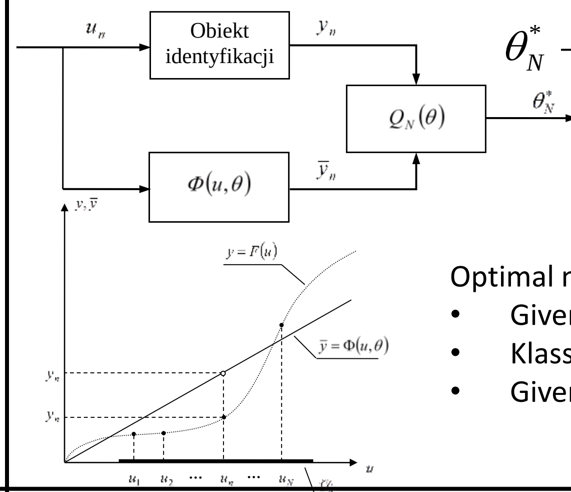

$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$
$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$




$$y_n = F(u_n,\theta),$$
$$n = 1, 2, \ldots, N$$
$$Y_N = \overline{F}(U_N,\theta)$$
$$\theta = \overline{F}^{-1}(U_N,Y_N) \overset{df}{=} \Psi_N(U_N,Y_N)$$

## Choice of the best model

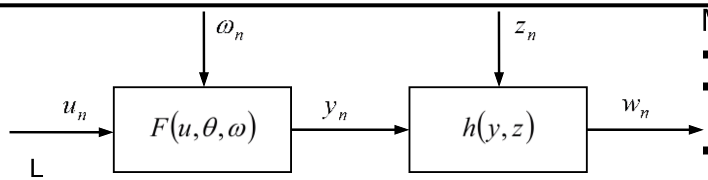
$$\bar{y} = \Phi(u,\theta_N^*)$$
$$\theta_N^* \to Q_N(\theta_N^*) = \min_{\theta} Q_N(\theta)$$
$$\theta_N^* = \Psi_N(U_N,Y_N)$$
$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$
$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$




Optimal model for:

- Given measurements sequences
- Klass of models
- Given performance index

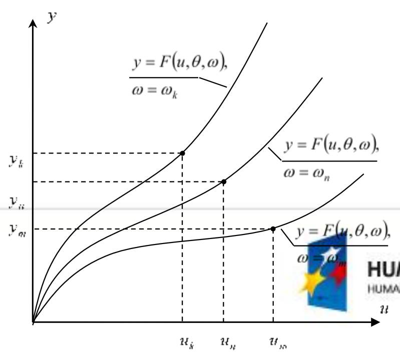
Losowy

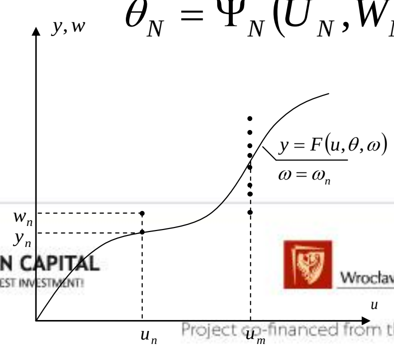


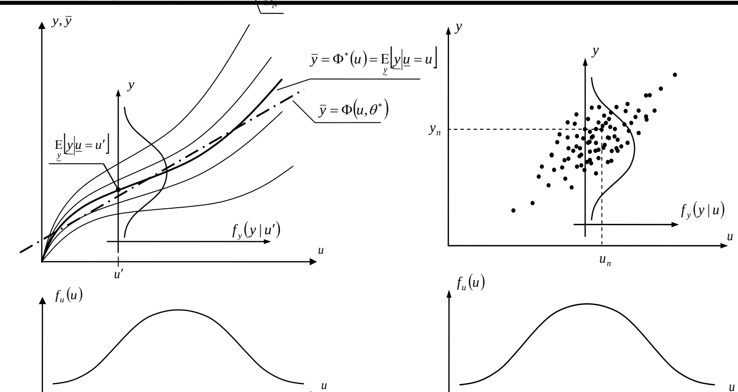

Methods:

- Least square
- Maximum likelihood
- Bayes'a

$$\theta_N = \Psi_N(U_N,W_N)$$



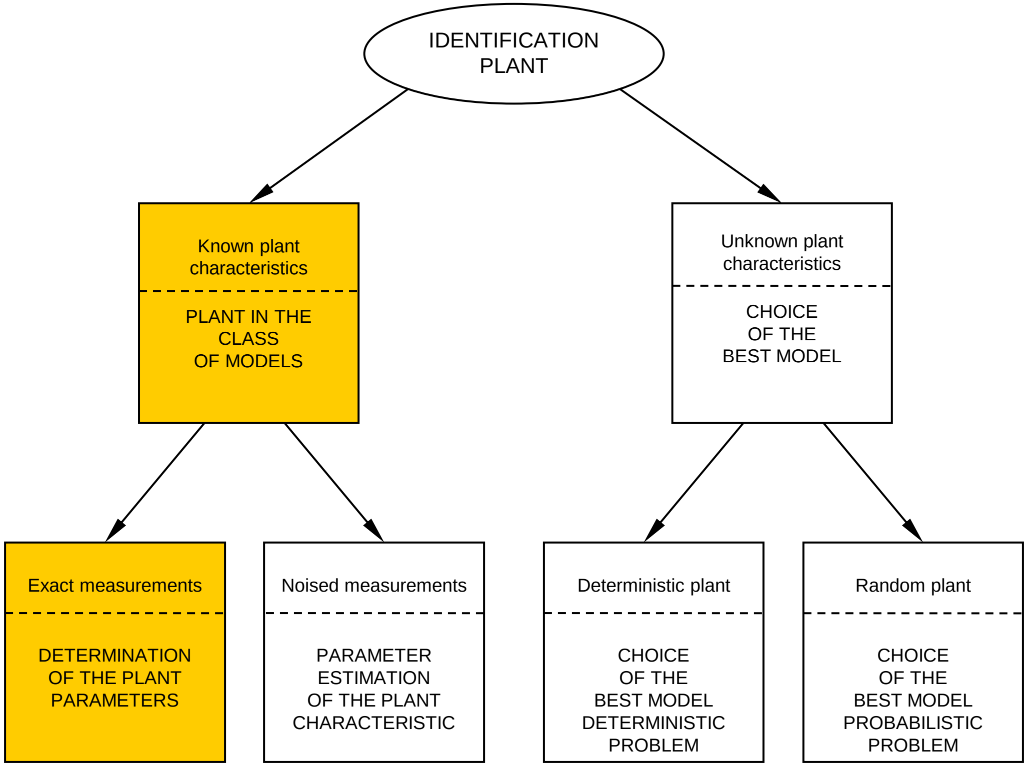

**Full probabilistic knowledge**

- First type regrsesion
- Second type regression

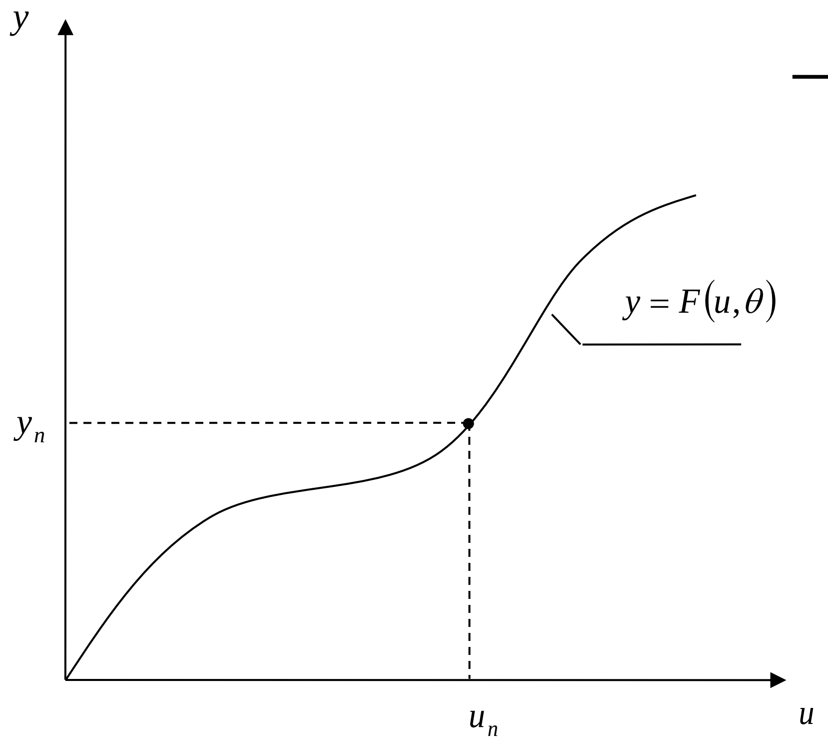
**Unknown probabilostic knowledge**

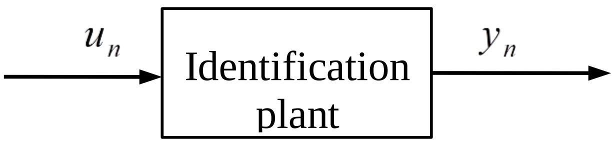
- Performance index estimatin
- Parameter estimation of the probability distribution
- Probabilit ditribution estimatoin

# Typical identification tasks



# Determination of the plant parameters (1)
## Exact Measurements



# Determination of the plant parameters (2)

- Problem formulation

<u>Static plant characteristic:</u> $y = F(u,\theta)$

$F$ – known function

$u$ – input vector $u \in \mathscr{U} \subseteq \mathscr{R}^S$ $\mathscr{U}$ – input domain

$y$ – output vector $y \in \mathscr{Y} \subseteq \mathscr{R}^L$ $\mathscr{Y}$ – output domain

$\theta$ – unknown vector of the plant characteristics parameters, $\theta \in \Theta \subseteq \mathscr{R}^R$

$\Theta$ – parameters domain

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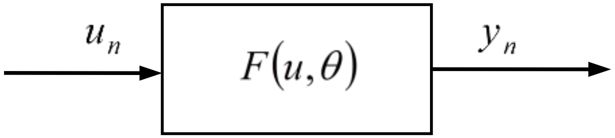
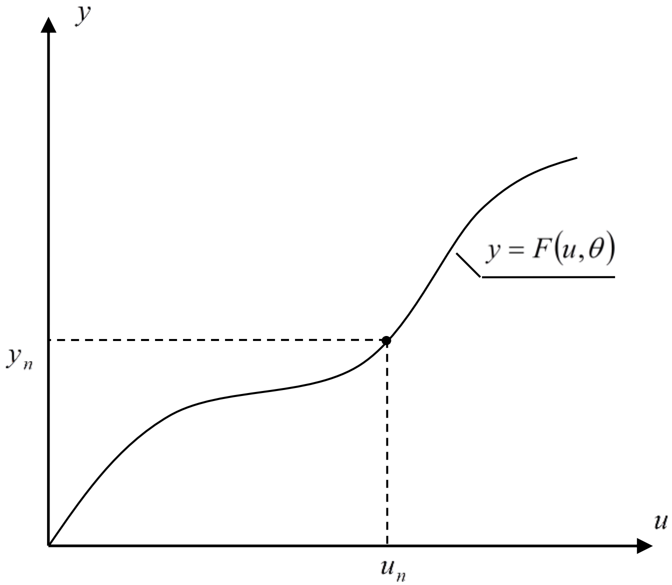
# Determination of the plant parameters (3)




Measurements:

$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$

$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$



# Determination of the plant parameters (4)

System of equations:

$$y_n = F(u_n, \theta), \quad n = 1, 2, \ldots, N$$

can be written

$$\begin{bmatrix} y_1 & y_2 & \cdots & y_N \end{bmatrix} = \begin{bmatrix} F(u_1,\theta) & F(u_2,\theta) & \cdots & F(u_N,\theta) \end{bmatrix}$$

For

$$\begin{bmatrix} F(u_1,\theta) & F(u_2,\theta) & \cdots & F(u_N,\theta) \end{bmatrix} \overset{df}{=} \overline{F}(U_N,\theta)$$

we can rewrite given set of equations:

$$Y_N = \overline{F}(U_N,\theta)$$

# Determination of the plant parameters (5)

Solution of $Y_N = \overline{F}(U_N, \theta)$ gives us identification algorithms:

$$\theta = \overline{F}^{-1}(U_N, Y_N) \overset{df}{=} \Psi_N(U_N, Y_N)$$

where:

$\overline{F}^{-1}$ - inverse function

$\Psi_N$ - identification algorithm

$$N \times L \geq R$$

# Determination of the plant parameters (6)

**Example**

Model

$$y = F(u,\theta) = \theta^T f(u)$$

where:

$$f(u) = \begin{bmatrix} f^{(1)}(u) \\ f^{(2)}(u) \\ \vdots \\ f^{(R)}(u) \end{bmatrix} \qquad \theta = \begin{bmatrix} \theta^{(1)} \\ \theta^{(2)} \\ \vdots \\ \theta^{(R)} \end{bmatrix}$$

and condition:

$$N = R$$

# Determination of the plant parameters (7)

For given model system of equations has the form:

$$y_n = \theta^T f(u_n), \quad n = 1, 2, \ldots, R$$

and can be rewritten:

$$Y_R \stackrel{df}{=} \begin{bmatrix} y_1 & y_2 & \cdots & y_R \end{bmatrix} = \begin{bmatrix} \theta^T f(u_1) & \theta^T f(u_2) & \cdots & \theta^T f(u_R) \end{bmatrix}$$

or

$$Y_R^T = \bar{f}^T (U_R)\theta$$

# Determination of the plant parameters (8)

where:

$$\bar{f}(U_R) \overset{df}{=} \left[ f(u_1) \quad f(u_2) \quad \cdots \quad f(u_R) \right]$$

and identification condition:

$$\det\left[\bar{f}^T(U_R)\right] \neq 0$$

Identification algorithm has the form:

$$\theta = \Psi_R(U_R, Y_R) = \left[\bar{f}^T(U_R)\right]^{-1} Y_R^T$$

# Determination of the plant parameters (9)

Special case: $f(u)=u$ , linear characteristic.

In this case system of equations has the form:

$$Y_R^T = U_R^T \theta$$

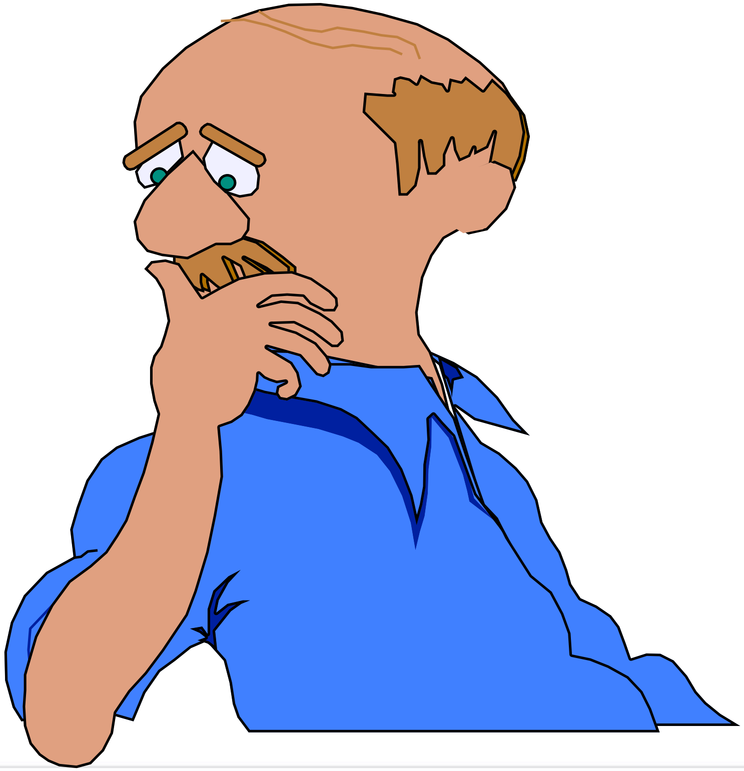
identification algorithm:

$$\theta = \Psi_N(U_R, Y_R) = \left[U_R^T\right]^{-1} Y_R^T$$

with condition:

$$\det\left[U_R\right] \neq 0$$

# Thank you for attention



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