Computer Science

Jerzy Świątek

Systems Modelling and Analysis

Choose yourself and new technologies

L.13a. Identification of dynamic plants

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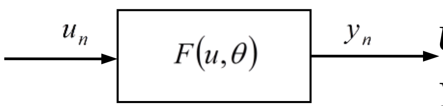
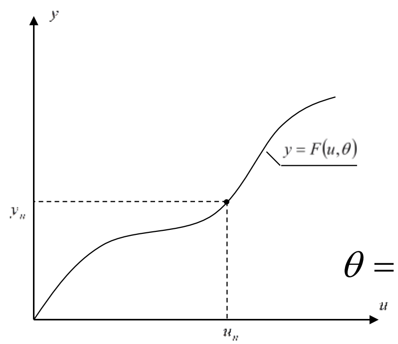
Plant in the class of model

Deterministyczny



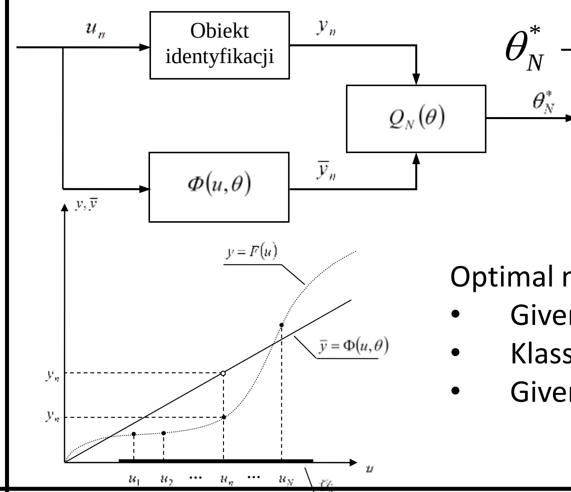
$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$
$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$



$$y_n = F(u_n,\theta),$$
$$n = 1, 2, \ldots, N$$
$$Y_N = \overline{F}(U_N,\theta)$$
$$\theta = \overline{F}^{-1}(U_N,Y_N) \overset{df}{=} \Psi_N(U_N,Y_N)$$

Choice of the best model

$$\overline{y} = \Phi(u,\theta_N^*)$$

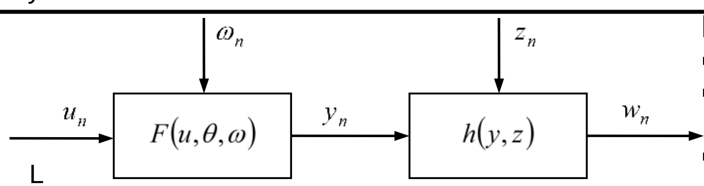


$$\theta_N^* \rightarrow Q_N(\theta_N^*) = \min_{\theta} Q_N(\theta)$$
$$\theta_N^* = \Psi_N(U_N,Y_N)$$
$$U_N = [u_1 \quad u_2 \quad \cdots \quad u_N],$$
$$Y_N = [y_1 \quad y_2 \quad \cdots \quad y_N]$$

Optimal model for:

- Given measurements sequences
- Klass of models
- Given performance index

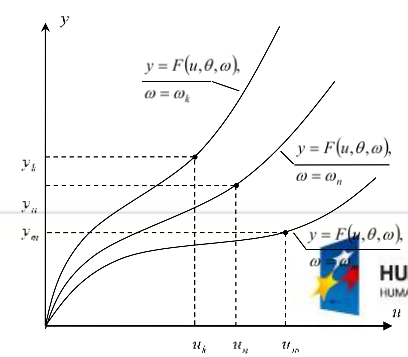
Losowy

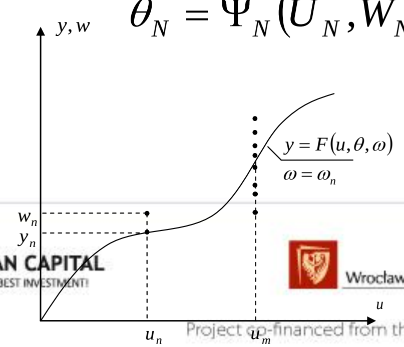


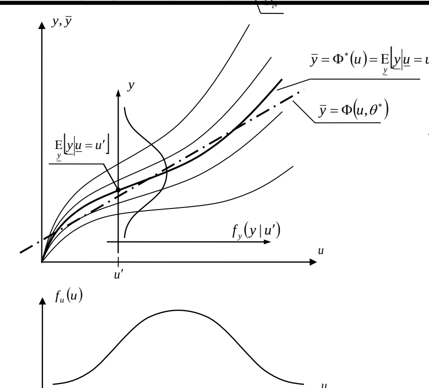
Methods:

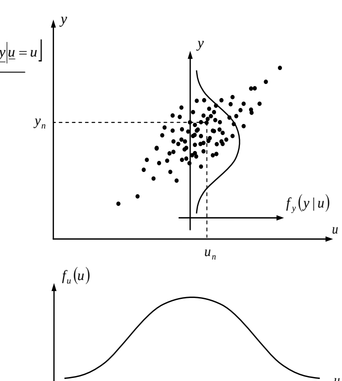
- Least square
- Maximum likelihood
- Bayes'a

$$\theta_N = \Psi_N(U_N,W_N)$$









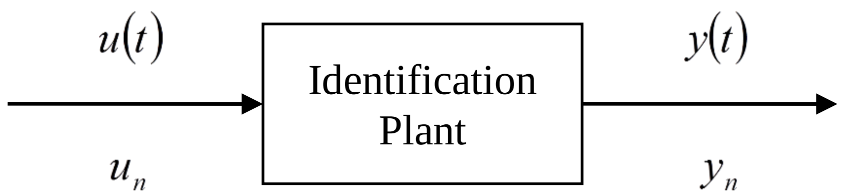
Full probabilistic knowledge

- First type regrsesion
- Second type regression

Unknown probabilostic knowledge

- Performance index estimatin
- Parameter estimation of the probability distribution
- Probabilit ditribution estimatoin

Identification of Dynamic Plants



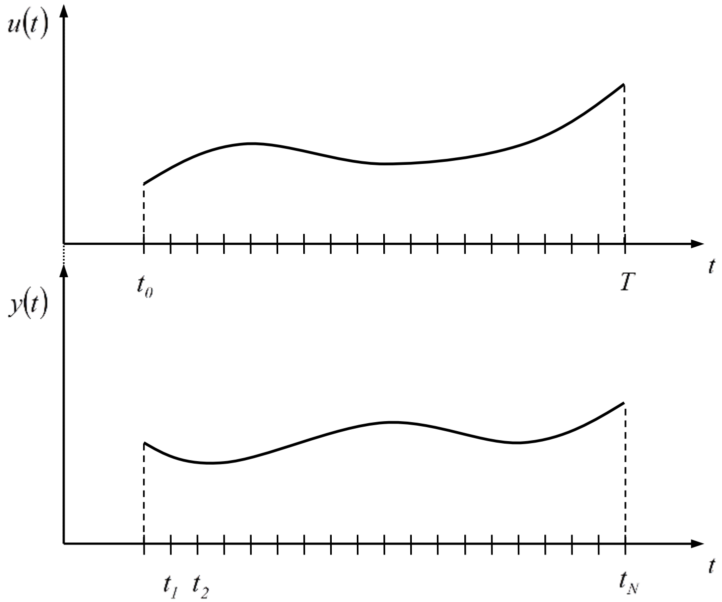
Descriptions:

- state variable;
- differential/difference equation;
- transfer functions: $K(s)$, $K(z)$;
- impulse response: $k_i(t)$, k_{in} ;
- step response: $h(t)$, h_n

Identification of Dynamic Plants

Continous Plant

For input signal $\{u(t)\}_{t_0}^{T}$ we measure respective output signal $\{y(t)\}_{t_0}^{T}$:

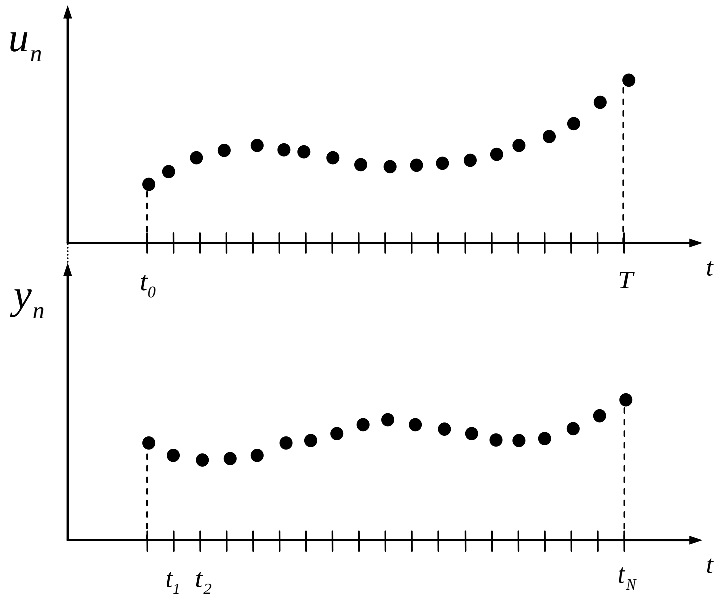


$$t_0 < t_1 < \ldots < t_N \leq T$$

$$\{u(t_n)\}_{n=0}^{N} \qquad \{y(t_n)\}_{n=0}^{N}$$

Identification of Dynamic Plants
Discrete Plant

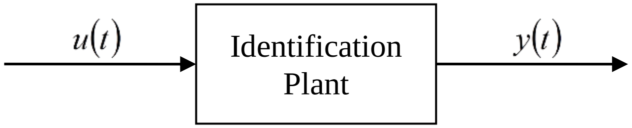
For input signal u_n we measure respective output signal y_n :



$$t_0 < t_1 < \ldots < t_N \leq T$$

$$\{u_n\}_{n=0}^{N} \qquad \{y_n\}_{n=0}^{N}$$

Identification of Dynamic Plants
Described by Differential equations



Let us assume that plant characteristic is known:

$$F\left(\frac{d^m y(t)}{dt^m}, \frac{d^{m-1} y(t)}{dt^{m-1}}, \ldots, \frac{dy(t)}{dt}, y(t); \frac{d^v u(t)}{dt^v}, \frac{d^{v-1} u(t)}{dt^{v-1}}, \ldots, \frac{du(t)}{dt}, u(t); \theta\right) = 0, \qquad m \geq v$$

and initial conditions w_0 are given.

The problem is to determine parameters θ.

Identification of Dynamic Plants
Described by Differential equations

If $u(t)$ is given **analytically**, e.g. $u(t)=\mathbf{1}(t)$, $u(t)=A\sin(\omega t)$;
then it is possible to work out solution analytically:

$$y(t)=\mathscr{F}\left(\{u(\tau)\}_{\tau=t_0}^{t},w_0;\theta\right)$$

$$y(t_n)=\mathscr{F}\left(\{u(\tau)\}_{\tau=t_0}^{t_n},w_0;\theta\right),\ n=1,2,\ldots,N$$

Solution of the above system of equations with respect θ gives identification algorithm.

Identification of Dynamic Plants
Described by Differential equations

Example

$$\frac{dy(t)}{dt} = \theta u(t)$$

Input signal:

$$u(t) = \mathbf{1}(t)$$

Initial condition:

$$y(0) = 0$$

Solution:

$$y(t) = \theta \int_o^t u(\tau) d\tau = \theta \int_0^t 1(\tau) d\tau = \theta t \qquad y(t_n) = \theta t_n \qquad \theta = \frac{y(t_n)}{t_n}$$

Identification of Dynamic Plants
Described by Differential equations

Approximate **numerical** solution:

Measurements:

$$\{u(t_n)\}_{n=0}^{N} \qquad \{y(t_n)\}_{n=0}^{N}$$

Numerical solution:

$$\tilde{y}(t_n,\theta) = \mathscr{F}_N\left(\{u(t_k)\}_{k=0}^{n}, w_0; \theta\right) \qquad y(t_n) \approx \tilde{y}(t_n,\theta)$$

$$\theta \approx \theta_N \rightarrow \min \underbrace{\sum_{n=0}^{N}\left(y(t_n) - \tilde{y}(t_n,\theta)\right)^2}_{B(\theta)} = \min \underbrace{\sum_{n=0}^{N}\left(y(t_n) - \mathscr{F}_N\left(\{u(t_k)\}_{k=0}^{n}, w_0; \theta\right)\right)^2}_{B(\theta)}$$

where:

$B(\theta)$ – error of numerical procedure

Identification of Dynamic Plants
Described by Differential equations

Calculations:

$$\theta_0 \rightarrow \tilde{y}(t_n, \theta_0) \rightarrow B(\theta_0)$$

$$\theta_1 \rightarrow \tilde{y}(t_n, \theta_1) \rightarrow B(\theta_1)$$

$$\vdots$$

$$\theta_K \rightarrow \tilde{y}(t_n, \theta_K) \rightarrow B(\theta_K)$$

$$\theta_K \approx \theta$$

K – number of steps of the numerical optimization methods

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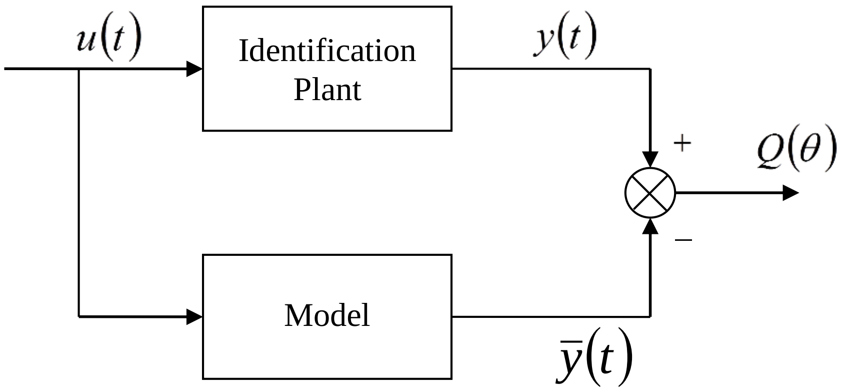
Identification of Dynamic Plants

Choice of the Best Model



$$\Phi\left(\frac{d^m\bar{y}(t)}{dt^m},\frac{d^{m-1}\bar{y}(t)}{dt^{m-1}},\ldots,\frac{d\bar{y}(t)}{dt},\bar{y}(t);\ \frac{d^v u(t)}{dt^v},\frac{d^{v-1}u(t)}{dt^{v-1}},\ldots,\frac{du(t)}{dt},u(t);\ \theta\right)=0$$

w_0 - initial conditions

Identification of Dynamic Plants

Choice of the Best Model

1. $u(t)$ is given **analytically**, e.g. $u(t)=\mathbf{1}(t)$, $u(t)=A\sin(\omega t)$;

Model:

$$\bar{y}(t,\theta)=\tilde{\Phi}\left(\{u(\tau)\}_{\tau=t_0}^{t}, w_0;\theta\right)$$

Performance index:

$$Q(\theta)=\int_{t_0}^{T}\left(y(t)-\bar{y}(t,\theta)\right)^2 dt=\int_{t_0}^{T}\left(y(t)-\tilde{\Phi}\left(\{u(\tau)\}_{\tau=t_0}^{t}, w_0;\theta\right)\right)^2 dt$$

Optimization problem:

$$\theta^* \rightarrow Q(\theta^*)=\min_{\theta\in\Theta} Q(\theta)$$

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Identification of Dynamic Plants
Choice of the Best Model

Example

Model: $$\frac{d\bar{y}(t)}{dt} = \theta u(t)$$

Input signal: $u(t) = \mathbf{1}(t)$

Initial condition: $y(0) = 0$

Solution:

$$\bar{y}(t) = \theta \int_{o}^{t} u(\tau)d\tau = \theta t$$

Performence index: $Q(\theta) = \int_{0}^{T} \left(y(t) - \theta t\right)^{2} dt$

$$\theta^{*} = \frac{\int_{0}^{T} y(t) t dt}{\int_{0}^{T} t^{2} dt}$$

Identification of Dynamic Plants

Choice of the Best Model

2. Discrete measurements of output signal: $\{u(t)\}_{t_0}^{T}, \{y(t_n)\}_{n=0}^{N}$;

Performance index:

$$Q_N(\theta) = \sum_{n=0}^{N} \left(y(t_n) - \tilde{\Phi}\left(\{u(\tau)\}_{\tau=t_0}^{t_n}, w_0; \theta \right) \right)^2$$

Optimization problem:

$$\theta_N^* \to Q_N(\theta_N^*) = \min_{\theta \in \Theta} Q_N(\theta)$$

Identification of Dynamic Plants
Choice of the Best Model

Numerical method:

Measurements: $\{u(t_n)\}_{n=0}^{N}$, $\{y(t_n)\}_{n=0}^{N}$;

Solution:

$$\tilde{\bar{y}}_N(t_n,\theta)=\tilde{\Phi}_N\left(\{u(t_k)\}_{k=0}^{n},w_0;\theta\right)$$

$$Q_N(\theta)=\sum_{n=0}^{N}\left(y(t_n)-\tilde{\bar{y}}_N(t_n,\theta)\right)^2=\sum_{n=0}^{N}\left(y(t_n)-\tilde{\Phi}_N\left(\{u(t_k)\}_{k=0}^{n},w_0;\theta\right)\right)^2$$

Identification of Dynamic Plants

Choice of the Best Model

We start from arbitrary taken θ_0 and then perform following calculations:

$$\theta_0 \rightarrow \bar{y}(t_n, \theta_0) \rightarrow Q_N(\theta_0)$$

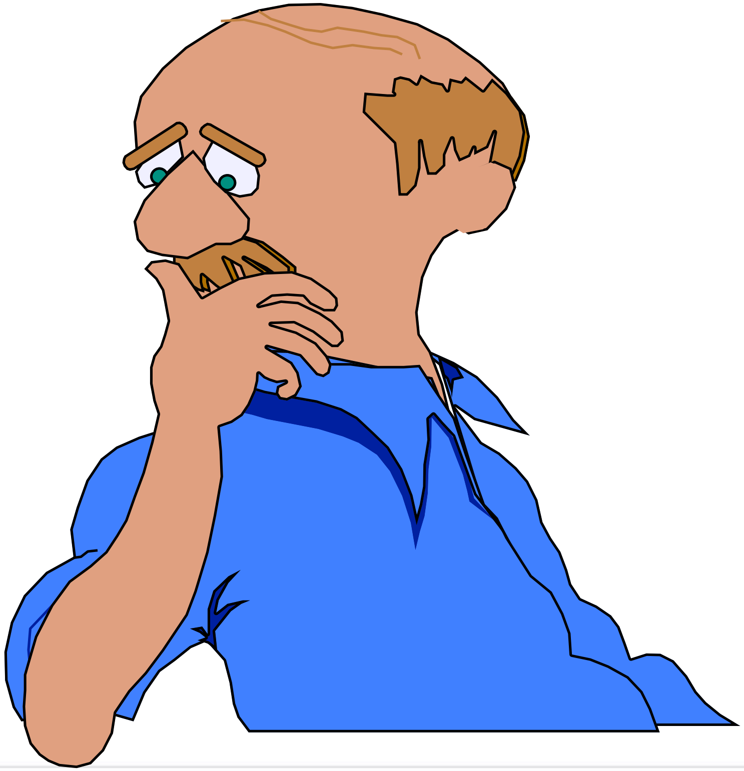
$$\theta_1 \rightarrow \bar{y}(t_n, \theta_1) \rightarrow Q_N(\theta_1)$$

$$\vdots$$

$$\theta_K \rightarrow \bar{y}(t_n, \theta_K) \rightarrow Q_N(\theta_K)$$

$$\theta_K^* \approx \theta_N^*$$

Thank you for attention



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